

1. In each case determine whether the given set is a subspace of R^3 . Explain your answer.

$$(i) H_1 = \left\{ \begin{bmatrix} r+s \\ 3r+2s-1 \\ r-t \end{bmatrix} \mid r, s, t \in R \right\} \quad (ii) H_2 = \left\{ \begin{bmatrix} a-b \\ b+c \\ 2a+c \end{bmatrix} \mid a, b, c \in R \right\}.$$

2. Which of the following sets is/are a subspace of R^3 .

$$U = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mid a-b+c=1 \right\}, \quad V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mid a+b=c \right\}$$

$$W = \left\{ \begin{bmatrix} a \\ 2 \\ c \end{bmatrix} \mid a=c \right\}, \quad H = \left\{ \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} \mid a+b=0 \right\}$$

- a) U and V b) V and W c) V and H d) U and W

3. Let $A = \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ -1 & 3 & 0 & 2 & -2 \\ 0 & 1 & 1 & 3 & 0 \end{bmatrix}$.

- a) Determine if the vector $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ is in $\text{Col}A$. Explain your reason.

- b) Determine if the vector $\begin{bmatrix} -12 \\ -4 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ is in $\text{Nul}A$. Explain your reason.

- c) Find bases for $\text{Col}A$ and $\text{Nul}A$.

- d) Find $\dim \text{Col}A$ and $\dim \text{Nul}A$.

- e) Verify the Rank Theorem.

4. You are given that $A = \begin{bmatrix} 1 & 3 & 2 & 1 & 3 \\ 2 & 7 & 5 & 1 & 8 \\ 1 & 3 & 2 & 2 & 4 \\ 2 & 9 & 7 & -1 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 2 & 1 & 3 \\ 0 & 1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a) Find bases for $\text{Col}A$ and $\text{Nul}A$.

b) Find $\dim \text{Col}A$ and $\dim \text{Nul} A$.

c) Determine if the vector $\begin{bmatrix} 4 \\ 6 \\ 7 \\ 2 \end{bmatrix}$ is in $\text{Col} A$. Explain your reason clearly.

d) Determine if the vector $\begin{bmatrix} 6 \\ -2 \\ -1 \\ -1 \\ 1 \end{bmatrix}$ is in $\text{Nul} A$. Explain your reason clearly.

5. Given that $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis for R^3 .

a) If $x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, find $[x]_{\mathcal{B}}$.

b) Use Part a) to evaluate $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}_{\mathcal{B}}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}_{\mathcal{B}}, \begin{bmatrix} -1 \\ 3 \\ 6 \end{bmatrix}_{\mathcal{B}}$.

6. Let H be a subspace of R^4 and $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}$ be a basis for H .

a) If $x = \begin{bmatrix} 1 \\ 3 \\ 2 \\ -1 \end{bmatrix}$, find $[x]_{\mathcal{B}}$ b) If $[y]_{\mathcal{B}} = \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix}$, find the vector y .